

ASYMPTOTIC STATE VECTOR COLLAPSE,
AND UNITARILY NONEQUIVALENT REPRESENTATIONS OF QED

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Abstract

The state vector evolution in the interaction of initial measured pure state with collective quantum system or the field with a very large number of degrees of freedom N is analysed in a nonperturbative QED formalism. As the example the measurement of the electron final state scattered on nucleus or neutrino is considered. In the nonperturbative field theory (QFT) the complete manifold of the system states is nonseparable i.e. is described by tensor product of infinitely many independent Hilbert spaces. The interaction of this system with the measured state can result in the final states which belong to different Hilbert spaces which corresponds to different values of some classical observables, i.e. spontaneous symmetry breaking occurs. Interference terms (IT) between such states in the measurement of any Hermitian observable are infinitely small and due to it the final pure states can't be distinguished from the mixed ones, characteristic for the state collapse. The evolution from initial to final system state is nonunitary and become formally irreversible in the limit of infinite time. The electromagnetic (e-m) bremsstrahlung produced in the electron scattering process contain the unrestricted number of soft photons which radiation flux become classic observable. Analogous processes which occurs in the second kind phase transitions in ferromagnetic and phonon excitations in crystal lattice are considered briefly.

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1. The problem of the state vector collapse description in Quantum Mechanics (QM) is still open despite the great efforts. The multiple proposals including such exotic as many worlds QM interpretations ,or collapse in the human brain constitute a long list, but in our opinion only microscopic dynamical models deserves serious analysis. Of them the most well developed are so called Decoherence models [1], despite they have serious internal consistency problems ,so called Environment Observables Paradox (EOP) [2]. In its essence it means that for any decoherence process exists at least one observable \hat{B} which expectation value will coincide with the pure final state expectations and differ from the predicted for the mixed state. Meanwhile Neeman proposed in 1985 that the solution of the problem can lay beyond the realm of standard QM and must be studied by the methods of modern nonperturbative QFT - dynamical theory with infinite number of degrees of freedom [3]. He considered the analogy between the collapse and the superconductor state evolution ,where spontaneous symmetry breaking of initial state result in appearance two different Hilbert spaces for quasiparticles which released with equal probability. In this approach EOP is resolved in a natural way ,due to the unobservability of interference between states from different spaces. Now this approach needs further detailization and application of some calculable models. In this paper we consider the model of this in the nonperturbative QED framework.

2. First we remind some nonperturbative QED results. In the perturbative QED framework for the electron $P_e \rightarrow P'_e$ scattered on some target the probability to emit photon with energy less than $\hbar\omega$ is proportional to $e^2 \ln \frac{P_e}{w}$ and so it grows unrestrictedly as $w \rightarrow 0$. In other words QED have running interaction constant $\alpha_r(\frac{P_e}{w})$ which diverge in this limit. It shows that perturbative methods are useless in this region where the number of simultaneously produced photons can be very large . The novel nonperturbative formalism which permit to calculate QED S-matrix correctly was developed in the last years [5]. It was proposed first for the case when electron evolution in momentum space can be regarded classically ,i.e. P_e, P'_e are eigenvalues of in,out states and electromagnetic current $J_\mu(x)$ is not the operator .but c-value which Fourier transform is equal to

$$J_\mu = J_\mu(k, p, p') = ie(\frac{p_\mu}{pk} - \frac{p'_\mu}{p'k}) \quad (1)$$

This formalism is correct if the recoil of radiated photons is small and doesn't change e final momentum, which is true for most cases.

Final photon state is calculated in the S-matrix approach by the action of operator T-product of interaction Hamiltonian density $\hat{H}_{em} = J_\mu(x)A_\mu(x)$ on the initial vacuum state [5]

$$|f\rangle = |\gamma^f\rangle = S(J)|0\rangle = \exp(i\phi(J) - V(J) + U(J))|0\rangle \quad (2)$$

where

$$V(J) = .5 \int d\tilde{k} J^*(k)J(k)$$

$$U(J) = i \int d\tilde{k} (J_\mu(k)a_\mu^+(k) - J_\mu^*(k)a_\mu(k))$$

where $d\tilde{k} = \frac{d^3k}{k_0}$. Here $\phi(J)$ is the pure phase, which is infinite for J_μ of (1) :

$$\phi(J) = \int \frac{d^4k J(k)J(-k)}{2(2\pi)^4 k^2}$$

This results in the divergent photon spectra

$$d\bar{N} = c \frac{d\omega}{\omega} = \frac{d\tilde{k}}{\hbar} |J(k)|^2 \quad (3)$$

well known in the classic theory. In the same time we get from (2)

$$|\langle f|0\rangle| = \exp(-\frac{\bar{N}}{2}) = 0$$

This case is equivalent to Bogolubov boson transformation of photon free field operators $a_\mu(k), a_\mu^*(k)$

$$b_\mu(k) = a_\mu(k) + iJ_\mu(k) \quad (4)$$

This transformation is nonunitary for J_μ given by (1) and the obtained final state doesn't belong to initial Fock space H_F . H_F is separable Hilbert space, i.e. can contain only states with $N < N_{max}$, where N_{max} is arbitrary large. In considered theory complete states manifold is nonseparable, i.e. described by the tensor product of the infinitely many Hilbert spaces H_i , where i index depends on the boson transformation constant $J_\mu(k)$ ($J_\mu(k) = 0$ for all k corresponds to H_F).

In the described QED approximation $|f\rangle$ will be generalised coherent state with infinite norm as can be seen from (2) [4]. This state is the cyclic vector in H_j and define the new vacuum state $|0\rangle_j$ from which we can construct all other vectors of H_j . Any Hermitian operator \hat{B} - observable acts only inside a single space $|\psi'\rangle_i = \hat{B}|\psi\rangle_i$, so that $\langle_i\psi|\hat{B}|g\rangle_j = 0$. So if the final state is the superposition of the states from several spaces $|f\rangle = |f_1\rangle_i + |f_2\rangle_j$, any interference terms (IT) between f_1, f_2 are unobservable and this states notified as disjoint states can't be distinguished from the mixture.

3. As the model of the state vector collapse we regard the weak scattering of the electron and some neutral particle (neutrino) ν with mass m_0 without assuming any classical properties of final states.

Amplitude M_w of weak vertex $e, \nu \rightarrow e', \nu'$ corresponds to finite cross-section and spherically symmetric distribution of e', ν' [4]:

$$M_w = \frac{G}{\sqrt{2}} J_{L\mu} J_{L\mu}^* = \bar{u}_e \gamma_\mu (1 + \gamma_5) u_\nu \bar{u}'_e \gamma_{\mu} (1 + \gamma_5) u'_{\nu} \quad (5)$$

Perturbative amplitude of the photon with polarisation e_μ to be radiated in this scattering, under condition that e final momentum is P' is:

$$M = M_w M(e \rightarrow e' \gamma) = M_w e J_\mu e_\mu \quad (6)$$

For general S-matrix calculation defined by $\hat{H}_i = \hat{H}_{em} + \hat{H}_w$ we use the smallness of weak interaction constant G which permit to calculate all the weak processes and

consequently e, ν momentums perturbatively. As previously we suppose the spectrum of P'_e is mainly defined by M_w neglecting photon recoil. In this approximation J_μ becomes the operator of fermion fields, but it conserve to commute with e-m field operators $\hat{A}_\mu(x)$ and in fact will define final e-me field state. The final system state is now completely nonclassical and is the the entangled product of e, ν and e-m field states

$$|f_w\rangle = \sum_{l=0} c_l |f_l\rangle = \sum_{l=1} c_l |e_l\rangle |\nu_l\rangle |\gamma_l^f\rangle + c_0 |e\rangle |\nu\rangle |0\rangle \quad (7)$$

where sum over l means integral over final e, ν momentums $p_l, p_{l\nu}$, $|\gamma_l^f\rangle = S(J_l)|0\rangle$, $J_l = J(k, p, p_l)$, c_0 is the rate of noninteracting particles, c_l is proportional to corresponding M_w . This formalism can be obtained in general form from the Low theorem as we discuss below. Note that this final state in general isn't coherent, which can have important consequences. The phases $\phi(J_l)$ are infinite, moreover their differences δ_{lm} are to be infinite also between disjoint final states f_l, f_m corresponding to different Hilbert spaces. Due to it in the limit $t = \infty$ this process is formally irreversible, because such infinite difference doesn't permit to define the relative phases of disjoint final states which must be operated as mixed ones. T-reflection of such final states and the consequent rescattering will result in the state completely different from the initial one. We don't discuss this effect at length, because in our calculations we'll not apply it directly. Note only that for classical case of (2) T-reflection will restore initial state but with some new arbitrary phase.

We want to measure in this layout if the act of scattering took place or the particles passed untouched and conserved their initial state. In the same time it will be the measurement of particles helicity, because at high energy $\sigma_L \gg \sigma_R$ for weak interactions. As the detector we can consider the single molecule D_2 which can dissociate in collision with the radiated photon with $E > E_d$ in 2 atoms D^* .

$$|f_{md}\rangle = \sum |f'_l\rangle = |D^*\rangle \sum_{l=1} c'_l |e_l\rangle |\nu_l\rangle |\gamma'_l\rangle + c_0 |D_2\rangle |\nu\rangle |e\rangle |0\rangle \quad (8)$$

where

$$|\gamma'_l\rangle = \int_{E_d} d\tilde{k} f(k) a(k) |\gamma_l^f\rangle$$

$f(k)$ is dissociation amplitude. In our study we neglect the final states $|f_l\rangle$ for which all photons have the energy $E < E_d$ regarding them as the detector inefficiency. Due to (4) $|\gamma'_l\rangle$ is the vector of the same space H_l to which $|\gamma_l^f\rangle = |\gamma^f\rangle_l$ belong. It follows that for any observable \hat{B} :

$$\langle e_l, \nu_l, D^* | \langle \gamma'_l | \hat{B} | 0 \rangle | e, \nu, D_2 \rangle = 0$$

So D final states are entangled with e-m field disjoint states which destroy IT.

Note that in practice direct \hat{B} IT observation is impossible even between single photon $|k\rangle$ and vacuum state as follows from Glauber theory of photocounting[6]. To reveal IT presence the special premeasurement procedure must be done, $|k\rangle$ must be reabsorbed by its source S and the interference of source states for some new observable B_s studied. The situation become even more complicated for nonunitary

evolution when B_s can be nonexistent. It certainly will be so for $|f_{md}\rangle$ states at $t = \infty$ due to discussed loss of relative phases between its parts $|f'_l\rangle$. Really if the phase differences δ_{lm} are infinite for disjoint final e-m field states, then their reabsorption will mean that phase loss is transferred to S states which become mixed. But we'll show this impossibility also for the experimental procedure performed at large, but finite time in the radiation interference formalism described in [4]. We don't consider at all the restoration of D_2 state, assuming it completely reversible and consider the rescattering of the state $|f_w\rangle$ of (7). We'll regard gedanken experiment where scattered e, ν are reflected by some very distant mirrors back to the interaction region where they rescatter again. This formalism is applied for charged current of e which passed through n consequent collisions with simple topological and casual structure. Then to calculate final state we can use (7) in which we take $J_l^s = \sum^n J_l^i$. But for the currents given by (1) we immediately get that the main part of it is equal to $J_\mu(k, p_{in}, p_{out}^l)$. In other words all intermediate steps are unimportant in agreement with Low theorem [4], which shows that infrared photon pole in any process is defined solely by the current calculated between asymptotic -in,-out states. As follows from (2) $\langle 0|S(J)|0\rangle$ amplitude of $|0\rangle$ restoration is nonzero only for $J_\mu^s = 0$ which means that e in and out momenta coincide, and from momentum conservation the same be true for ν . So we must calculate the probability P of weak process $i \rightarrow n'_l \rightarrow i$, where n'_l are all possible intermediate states, which we suppose have the same spectra as final states. Its calculation is simplified by the spherical symmetry of weak scattering (5), so that neglecting diffraction we can omit sum over intermediate states and obtain:

$$P = \frac{\int |M_w(n'_1 \rightarrow i)|^2 do_f}{\int |M_w(n'_1 \rightarrow n_o)|^2 do_f} = 0$$

where n'_1 is arbitrary intermediate state, n_o is sum over final states. o_f is phase space of final e states which is isomorphic to spherical surface with $r = 1$ with nearly constant density of final states on it. Then restoration of initial state corresponds to a single point r_i on this surface. Each infinitely close point to r_i corresponds to another Hilbert space generating infinite number of soft photons. So the zero probability of initial state restoration obtains simple geometrical interpretation in which r_i is singular point in phase space which we can omit without changing any physical result.

The same effects can be expected for the coulomb e scattering for which bremsstrahlung were most often studied, but due to long range of this force the analysis will be more intricate.

4. We've shown that final states of $e - \nu$ scattering asymptotically reveal the properties of the mixed state i.e. perform the collapse. This doesn't seem a surprise, because the classical features of electron bremsstrahlung states were stressed often, but to our knowledge the final e states interference never was analysed. [5]. In the alternative approach developed by Buchholz this classical properties results from electric field flow conservation constituting additional superselection rule [7]. This rule based on Gauss law assumes that Lorentz symmetry for the electron is spontaneously broken. It supposes that the collapse is induced primarily not by the

huge number of radiated photons but the long range properties of vector potentials which doesn't permit the superpositions of charged states with different velocities at the infinite time limit. It must result in some complications in the description of localised charged states structure and evolution which will be analysed elsewhere.

The real detectors are localised solid objects to which this formalism doesn't applicable directly. But the general QFT analysis was very succesful for the solid state phenomena description ,and so we'll scatch here the possible framework for the collapse models. The simple model in which collapse is induced by the by the 2nd order phase transition in ferromagnetic was given in [8] and we'll descuss here its possible developments. It's well known that the transition from the individual particles (atoms) to quasyparticles - phonons, magnons in the infinite media can be described as boson transformation analogous to (4) [9]. The resulting quasiparticles are massless and the excitation spectra have no gap i.e. infrared divergent. Despite the media is electrically neutral this quanta readily interact with e-m field, so any excitation of this system in the vacuum can be relaxated by the infrared divergent radiation (4).

This idea is also applicable for the finite system if its surface is regular and transparent for radiation. This surface can be regarded as the topological defect with infinite number of degrees of freedom which result in a special kind of boson condensation in its volume[10]. So the system states manifold can be unitarily nonequivalent and the resulting quasyparticles spectra is infrared divergent. The same we can expect for the relaxation photons radiated through the crystall surface. We consider as the detector the idealised crystall which can be excited by the high energy neutral or charged particle. We take the initial state $|i\rangle$ to be the superposition of 2 localised states with the trajectories x_1, x_2 passing through and beyond the crystall, which has to be measured. So analogous to (7) the final state is the superposition of initial vacuum and some new excited state. The state $|x_1\rangle$ can kick out or shift several atoms producing lattice dislocation and excitation. So its final state after lattice relaxation will include infinite number of soft photons which constitute the new vacuum completely ortogonal to initial one.

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